

Geometry 1-4 Study Guide: Pairs of Angles (pp 28-31)

Attendance Problems. Simplify each expression.

1. $90 - (x + 20)$

2. $180 - (3x - 10)$

Write an algebraic expression for each of the following.

3. 4 more than twice a number.

4. 6 less than half a number

- I can identify adjacent, vertical, complementary, and supplementary angles.
- I can find measures of pairs of angles.

Vocabulary		
adjacent angles	linear pair	complementary angles
supplementary angles	vertical angles	

Common Core: CC.9-12.G.CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometry software, etc.).

Many pairs of angles have special relationships. Some relationships are because of the measurements of the angles in the pair. Other relationships are because of the positions of the angles in the pair.

Teacher: Today we are going to learn about complementary angles.

Student: Does that mean the angles are nice to each other?

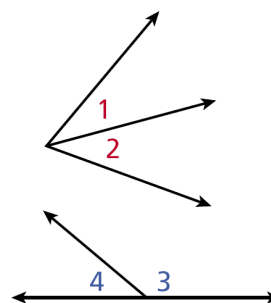
"Flight Reservation systems decide whether or not you exist. If your information isn't in their database, then you simply don't get to go anywhere." -- Arthur Miller

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Pairs of Angles

Adjacent angles are two angles in the same plane with a common vertex and a common side, but no common interior points. $\angle 1$ and $\angle 2$ are adjacent angles.

A **linear pair** of angles is a pair of adjacent angles whose noncommon sides are opposite rays. $\angle 3$ and $\angle 4$ form a linear pair.

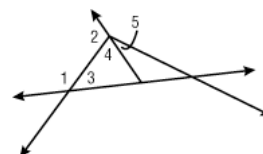


Video Example 1. Tell whether the angles are only adjacent, adjacent and form a linear pair, or not adjacent.

A. $\angle 1$ & $\angle 2$

B. $\angle 4$ & $\angle 5$

C. $\angle 1$ & $\angle 3$



1 Identifying Angle Pairs

Tell whether the angles are only adjacent, adjacent and form a linear pair, or not adjacent.

A $\angle 1$ and $\angle 2$

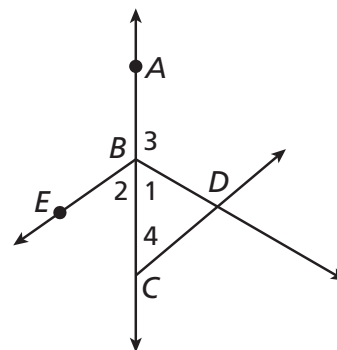
$\angle 1$ and $\angle 2$ have a common vertex, B , a common side, \overrightarrow{BC} , and no common interior points. Therefore $\angle 1$ and $\angle 2$ are only adjacent angles.

B $\angle 2$ and $\angle 4$

$\angle 2$ and $\angle 4$ share \overrightarrow{BC} but do not have a common vertex, so $\angle 2$ and $\angle 4$ are not adjacent angles.

C $\angle 1$ and $\angle 3$

$\angle 1$ and $\angle 3$ are adjacent angles. Their noncommon sides, \overrightarrow{BC} and \overrightarrow{BA} , are opposite rays, so $\angle 1$ and $\angle 3$ also form a linear pair.



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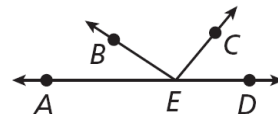
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Example 1. Tell whether the angles are only adjacent, adjacent and form a linear pair, or not adjacent.

A. $\angle AEB$ & $\angle BED$

B. $\angle AEB$ & $\angle BEC$

C. $\angle DEC$ & $\angle AEB$

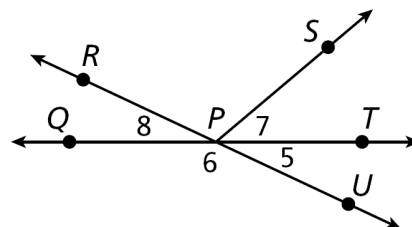


Guided Practice. Tell whether the angles are only adjacent, adjacent and form a linear pair, or not adjacent.

5. $\angle 5$ & $\angle 6$

6. $\angle 7$ & $\angle SPU$

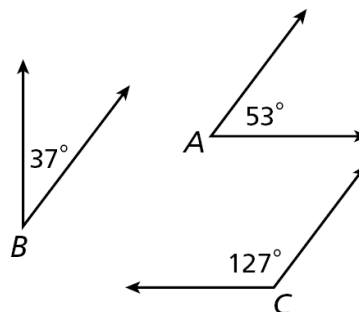
7. $\angle 7$ & $\angle 8$



Complementary and Supplementary Angles

Complementary angles are two angles whose measures have a sum of 90° .
 $\angle A$ and $\angle B$ are complementary.

Supplementary angles are two angles whose measures have a sum of 180° .
 $\angle A$ and $\angle C$ are supplementary.



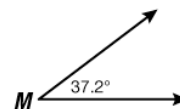
You can find the complement of an angle that measures x° by subtracting its measure from 90° , or $(90 - x)^\circ$.

You can find the supplement of an angle that measures x° by subtracting its measure from 180° , or $(180 - x)^\circ$.

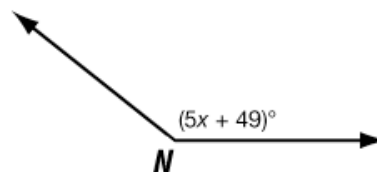
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Video Example 2. Find the measure of each of the following.

A. Find the measure of the complement of $\angle M$.



B. Find the measure of the supplement of $\angle N$.



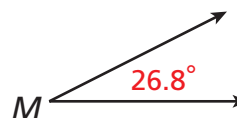
2 Finding the Measures of Complements and Supplements

Find the measure of each of the following.

A complement of $\angle M$

$$(90 - x)^\circ$$

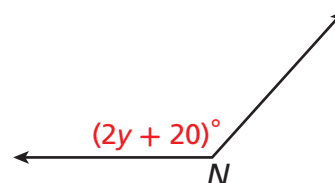
$$90^\circ - 26.8^\circ = 63.2^\circ$$



B supplement of $\angle N$

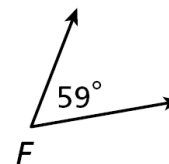
$$(180 - x)^\circ$$

$$180^\circ - (2y + 20)^\circ = 180^\circ - 2y - 20$$
$$= (160 - 2y)^\circ$$

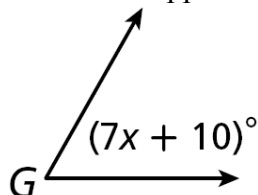


Example 2. Find the measure of each of the following.

A. The complement of $\angle F$.



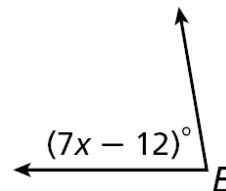
B. The supplement of $\angle G$.



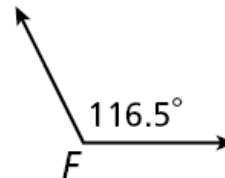
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Guided Practice. Find the measure of each of the following.

8. The complement of $\angle E$.



9. The supplement of $\angle F$.



Video Example 3: Twice an angle measures 9° more than the measure of its complement. Find the measure of its complement.

3

Using Complements and Supplements to Solve Problems

An angle measures 3 degrees less than twice the measure of its complement. Find the measure of its complement.

Step 1 Let $m\angle A = x^\circ$. Then $\angle B$, its complement, measures $(90 - x)^\circ$.

Step 2 Write and solve an equation.

$$m\angle A = 2m\angle B - 3$$

$$x = 2(90 - x) - 3$$

$$x = 180 - 2x - 3$$

$$x = 177 - 2x$$

$$\begin{array}{r} + 2x \quad \quad + 2x \\ \hline \end{array}$$

$$3x = 177$$

$$\begin{array}{r} 3x = 177 \\ \hline 3 \quad \quad 3 \end{array}$$

$$x = 59$$

Substitute x for $m\angle A$ and $90 - x$ for $m\angle B$.

Distrib. Prop.

Combine like terms.

Add $2x$ to both sides.

Simplify.

Divide both sides by 3.

Simplify.

The measure of the complement, $\angle B$, is $(90 - 59)^\circ = 31^\circ$.

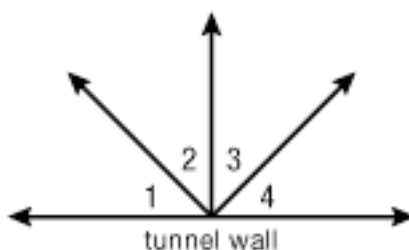
Example 3: An angle is 10° more than 3 times the measure of its complement. Find the measure of the complement.

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10. Guided Practice. An angle's measure is 12° more than $\frac{1}{2}$ the measure of its supplement. Find the measure of the angle.

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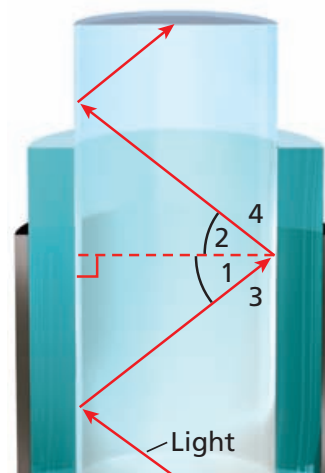
Video Example 4. Light passes through a tunnel and reflects off the walls in such a way that $\angle 2 \cong \angle 3$, $\angle 1$ & $\angle 2$ are complementary, and $\angle 3$ & $\angle 4$ are complementary. If $m\angle 4 = 52^\circ$, find $m\angle 1$, $m\angle 2$, & $m\angle 3$.



4 Problem-Solving Application

Light passing through a fiber optic cable reflects off the walls in such a way that $\angle 1 \cong \angle 2$. $\angle 1$ and $\angle 3$ are complementary, and $\angle 2$ and $\angle 4$ are complementary.

If $m\angle 1 = 38^\circ$, find $m\angle 2$, $m\angle 3$, and $m\angle 4$.



1 Understand the Problem

The answers are the measures of $\angle 2$, $\angle 3$, and $\angle 4$.

List the important information:

- $\angle 1 \cong \angle 2$
- $\angle 1$ and $\angle 3$ are complementary, and $\angle 2$ and $\angle 4$ are complementary.
- $m\angle 1 = 38^\circ$

2 Make a Plan

If $\angle 1 \cong \angle 2$, then $m\angle 1 = m\angle 2$.

If $\angle 3$ and $\angle 1$ are complementary, then $m\angle 3 = (90 - 38)^\circ$.

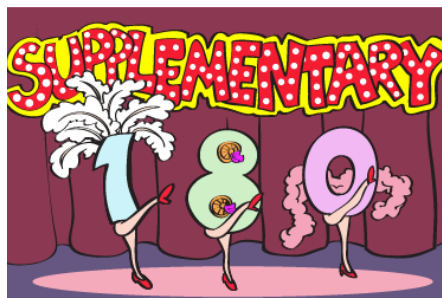
If $\angle 4$ and $\angle 2$ are complementary, then $m\angle 4 = (90 - 38)^\circ$.

3 Solve

By the Transitive Property of Equality, if $m\angle 1 = 38^\circ$ and $m\angle 1 = m\angle 2$, then $m\angle 2 = 38^\circ$. Since $\angle 3$ and $\angle 1$ are complementary, $m\angle 3 = 52^\circ$. Similarly, since $\angle 2$ and $\angle 4$ are complementary, $m\angle 4 = 52^\circ$.

4 Look Back

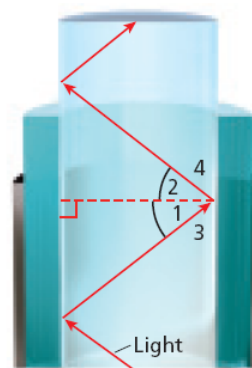
The answer makes sense because $38^\circ + 52^\circ = 90^\circ$, so $\angle 1$ and $\angle 3$ are complementary, and $\angle 2$ and $\angle 4$ are complementary. Thus $m\angle 2 = 38^\circ$, $m\angle 3 = 52^\circ$, and $m\angle 4 = 52^\circ$.



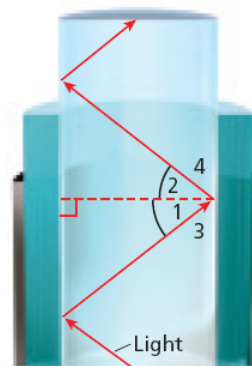
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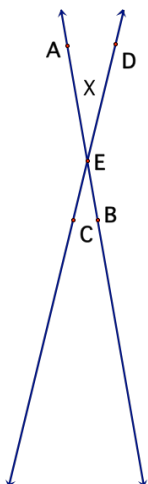
Example 4. Light passing through a fiber optic cable reflects off the walls of the cable in such a way that $\angle 1 \cong \angle 2$, $\angle 1$ and $\angle 3$ are complementary, and $\angle 2$ and $\angle 4$ are complementary. If $m\angle 1 = 47^\circ$, find $m\angle 2$, $m\angle 3$, and $m\angle 4$.



11. Guided Practice. Suppose $m\angle 3 = 27.6^\circ$. Find $m\angle 1$, $m\angle 2$, & $m\angle 4$.



12. Now consider the diagram which shows \overline{AB} & \overline{CD} intersecting at E. If $x = 23^\circ$, find $m\angle AEC$, $m\angle DEB$, & $m\angle CEB$. Show some work.



13. Based on your work from the problem 12, which angle has the same measure as $\angle AED$?

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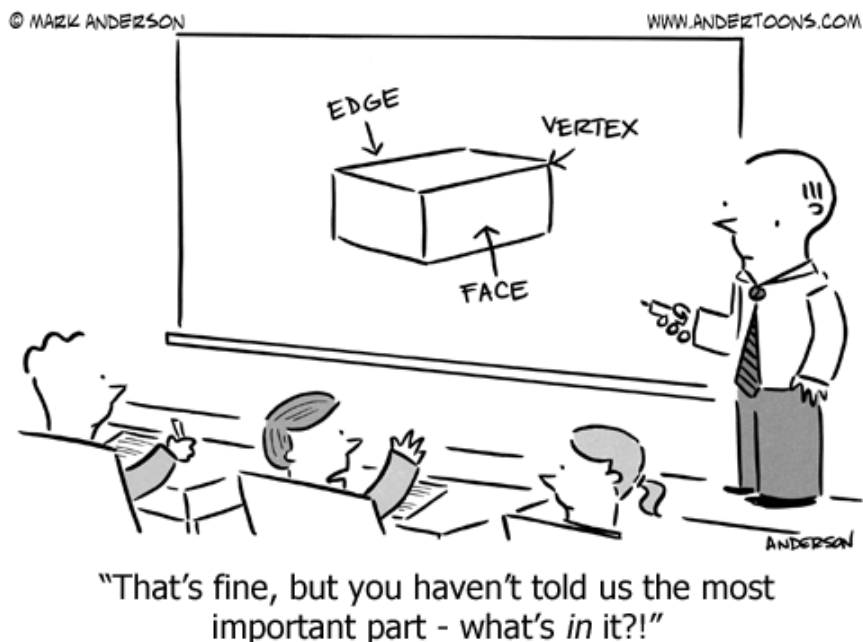
14. When two lines intersect the angles that lie on opposite sides of the intersection point are called **vertical angles** (that is a shortened name, the full name is vertically opposite angles). For example in the diagram for question 12, $\angle AED$ & $\angle CEB$ are vertical angles. Find another pair of vertical angles.

Travis noticed that the vertical angles in questions 12 and 13 have equal measures and wondered if other pairs of vertical angles also have equal measures.

15. Again, refer to the diagram for question 12, this time assume $x = 54^\circ$. Find $m\angle CEB$. Show some work.

16. Based on your observations, write a **conjecture** (a statement based on an educated guess that is unproven). Complete the following conjecture: Vertical angles

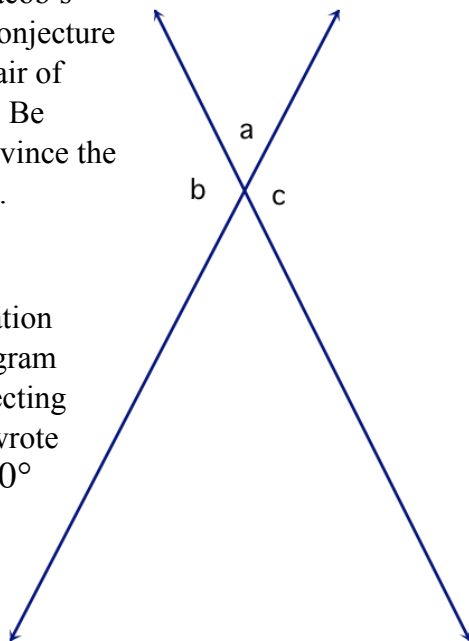
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When Jacob answered question 16, he wrote the conjecture: “*Vertical angles have equal measure.*” (Remember that a conjecture is an educated guess that has not been proven.)

17. Do you think Jacob’s vertical angle conjecture holds for *any* pair of vertical angles? Be prepared to convince the rest of the class.



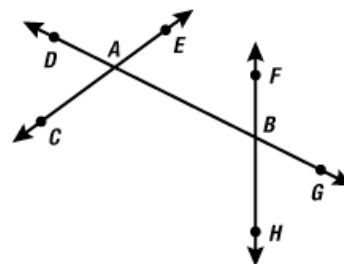
18. Jacob’s explanation included the digram showing intersecting lines. He then wrote that $a + b = 180^\circ$ and $a + c = 180^\circ$. Are these statements true? Why?

19. How can you use Jacob’s statements in question 18 to prove that vertical angles always have the same measure?

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20. Once a conjecture is proven to be true, it is referred to as a **theorem**. Proving that vertical angles are always congruent in question 19 changed this conjecture into a theorem that can now be used in the later problems without needing to reprove it again. Discuss with your table the differences between a conjecture and a theorem and write down your ideas about the differences.

Video Example 5. Name four pairs of vertical angles in the diagram.

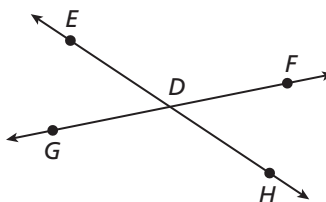


5 Identifying Vertical Angles

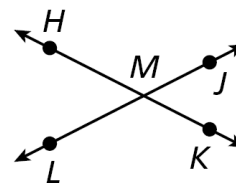
Name one pair of vertical angles.
Do they appear to have the same measure?
Check by measuring with a protractor.

$\angle EDF$ and $\angle GDH$ are vertical angles
and appear to have the same measure.

Check $m\angle EDF \approx m\angle GDH \approx 135^\circ$.



Example 5. Name the pairs of vertical angles.



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21. Angle Relationships. Describe each of the angle relationships you learned about in this study guide. Include a diagram, a description of the angles and what you know about the relationship. For example, are they angles always equal. Do they have a special sum?

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Ready to Go On Section 1A pretest & posttests